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[ALLOCATION OF ENTROPY AS CONVEX CONES]

It has been mathematically determined that allocation of entropy are convex cones for bipartite and tripartite quantum system but there is lack of such pictorial view, this project attempts to find 3D view of convex cones for bipartite and tripartite quantum system

Allocation of Entropy as Convex Cone for Bipartite and Tripartite Quantum System

Abstract

Central to the Quantum Information Theory is the concept of Entropy and the Inequalities Theorem of Weak Subaddivity and Strong Subaddivity. While Entropy indicates amount of information content of a system or a composite system, the inequality theorems indicates changes in information contained in each individual system and composite system. Bipartite and Tri-partite systems are composite system for which the weak and strong subadditivity theorem holds. These systems have been extensively studied and one of the result obtained is that the allocation of entropy governed by the inequalities theorem are convex cones. It is known that entropy is a concave function of its inputs, which is probability distribution in the form of density matrices. The lack of geometrical picture of the allocation of entropy forming convex cones governed by the inequality theorem is the subject of this project. Bipartite and tripartite systems are investigated, set of entropies obtained, and with the inequalities theorem 2D and 4D convex cones are obtained. Malab is used for obtaining the geometrical picture of the cones.

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1. Introduction

Quantum Information Theory is an emerging field in the Information Communication Technology (ICT) which lies at the intersection of quantum mechanics, information and communication theory and computer science. It promises improvements over classical communication- faster and more efficient information processing capability, secure communication that cannot be compromised through quantum cryptography and distributed key distribution and quantum teleportation that allows unique kind of communication between source and receiver. Much of the advancement achieved however are mathematical results rather than physical realization but nevertheless subtle practical demonstration of the quantum system has been verified and used in commercial devices. Among the central mathematical relation and framework of Quantum Information Theory are Entropy and the Inequalities of Quantum Information Theory around which this project is based on.

1.1 Entropy

Central to quantum information theory is the concept of Entropy. In classical information theory, Shannon's entropy is used and in quantum information theory Von Neumann Entropy is used. These are given below:

Shannon's Entropy is given by the relation,

$$H(X) = -\sum P_x \log P_x \quad (1)$$

where, X is a random variable and P_x denotes probability distribution of the random variable

The Entropy in QIT is the Von Neumann Entropy which is given by the relation,

$$S(Q) = -Tr(Q \log Q) \quad (2)$$

Or

$$S(Q) = -\sum_i \lambda i \ \log \lambda i$$

where, Q is the density matrix of quantum state of a system or combined system and Tr denotes trace of matrix and λ denotes the eigen values of the density matrix and the summation is over all the eigen values 'i'. The Von Neumann Entropy is a positive quantity and is a concave function of its input(s).

1.2 Convex Set, Convex Function and Convex Cone

A function 'f' is said to be a convex function if it satisfies the following inequalities;

$$f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda) f(y) \qquad \text{for all } 0 \leq \lambda \leq 1 \tag{3}$$

where, x and y are any real numbers

A set is called a Cone if for every $x \in C$ and $\lambda x \geq 0$, λx belongs to set C.

That is if,

$$\lambda x \in C \tag{4}$$

A set C is a convex cone if it is both convex and a cone.

That is, if C is a set of points in R^k, then E is a convex cone if

1. $for\ every\ x \in C\ and\ every\ real\ \lambda \geq 0,\ \lambda X \in C$
2. $for\ every\ x \in C\ and\ y \in C, and\ every\ real\ 0 \leq \lambda \leq 1,\ \lambda x + (1-\lambda)y \in C$

1.3 Allocation of Entropy

If Q denote the density matrix of a quantum state having n parts (1, 2….n).
For $I \subseteq N = \{1,2 \ldots n\}$, if $Q_I = Tr_{N/I}(Q)$ denote the density matrix of the state comprising those parts i such that $i \in I$, and if $S(Q_I)$ denote the Von Neumann Entropy of the state Q_I. Then the collection of $v = 2^n$ numbers $\{S(Q_I\}$ or set of entropy is called allocation of entropy for Q in the vector space R^v.

1.4 Basic Inequalities of Quantum Information Theory

Fundamental inequalities theorems in quantum information theory are monotonicity and subadditivity theorems. These theorems have established fundamental properties of quantum states in quantum information theory. They govern the operations and processing properties within a quantum system.

If Q_1, Q_2, Q_3 represents the density matrices of any three quantum states 1,2 and 3 then, Weak Monotonicity Inequality is expressed as,

$$\mathbf{S(Q_1) + S(Q_2) \leq S(Q_{13}) + S(Q_{23})} \quad (5)$$

Strong Subadditivity Inequality is expressed as,

$$\mathbf{S(Q_{123}) + S(Q_3) \leq S(Q_{13}) + S(Q_{23})} \quad (6)$$

If $Q_3=1$ that is the density matrix of quantum state is unity, then the above relation reduces to Weak Subadditivity as,

$$\mathbf{S(Q_{12}) \leq S(Q_1) + S(Q_2)} \quad (7)$$

where, $S(Q_i)$ represents Von Neumann Entropy of any quantum states i, and $S(Q_{ij})$ or $S(_{Q_{ijk})}$ represents the Joint Von Neumann Entropy of quantum states i, j or i, j and k respectively.

2. Problem Statement

How will the mathematical description of different types of allocation of entropy for varying degree of partite system (n) look like in 2D and 3D plot? It is known and has been proved that the different types of allocation of entropy governed by inequality are convex cones. But there is lack of corresponding pictorial, geometrical view of such theoretical treatment of inequalities. This is the core area that this project will address.

3. Objective of the Project

The purpose of this project is to find out pictorial, geometrical, physical interpretation of allocation of entropy bipartite and tripartite quantum system (n=2 for bipartite system and n=3 for tripartite system). This should provide better knowledge and understanding of the inequalities of quantum information as allocation of entropy. Taking as reference the IEEE Transaction paper authored by Nicholas Pippenger with the help of Matlab, the objective is to plot those set of allocations. Through the geometry, possible specific pattern will be investigated.

The project will give valuable insight into the geometry of different types of allocation of entropy for various degree of partite system (n = 2 and n=3). This should in turn give more understanding of the higher degree of partite system (n≥4) and hence the associated inequalities of the system under consideration. Further this should provide better understanding of geometric pattern of allocation of entropies.

4. Methodology

The Von Neumann entropies for bipartite and tripartite quantum system will be calculated using Matlab. The matlab required functions for calculating Von Neumann entropy and function for plotting 2D, 3D graphs are available on the web and Matlab.

To realize the allocation of entropy, which should be consistent with the inequality theorems for bipartite and tripartite system, three different systems setting are explored.

1. Bipartite System
   Two types of System are considered here
   a. Mixed System of |0> and |1>
   b. Mixed System of |0> and |+>
2. Tripartite System

1. Bipartite System

a. Mixed System of |0> and |1>
   The quantum state of system 'A' is |0> and the quantum state of system 'B' is |1>. The density matrix of system A and system B are denoted by Q_A and Q_B respectively. That is,

$$Q_A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \text{ and } Q_B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

b. Mixed System of |0> and |+>
   The quantum state of system 'A' is |0> and the quantum state of system 'B' is |+>. The density matrices for system A and B are thus,

$$Q_A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \text{ and } Q_B = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$

2. Tripartite System
   Let A, B, C denote the three system, then the following quantum states with density matrix will be taken for tri-partite system purpose

$$Q_A = \begin{pmatrix} 3/4 & 0 \\ 0 & 1/4 \end{pmatrix}, Q_B = \begin{pmatrix} 9/10 & 0 \\ 0 & 1/10 \end{pmatrix} \text{ and } Q_C = \begin{pmatrix} 1/2 & 1/4 \\ 1/4 & 1/2 \end{pmatrix}$$

The entropies for each system will be evaluated to obtain allocation of entropy (set of entropy values). Then inequalities theorems will be tested in each case. This allows checking the consistency of the calculated data. Finally, the sets of entropy values will be plotted in Matlab to obtain Convex Cones as per the project objective.

5. Results

The following are tables containing the values of density matrices and quantum entropies for the two bipartite and tripartite systems discussed above.

Table 1: Calculation of Density Matrices and Entropies for Bipartite System composed of |0> and |1> states

Sr. No	% of State 'A' or '|0>'	% of State 'B' or '|1>'	Q_A	Q_B	Q_{AB}	S (Q_A)	S (Q_B)	S (Q_{AB})
1	0	100	$0 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$1\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	0	0	0
2	5	95	$1/20 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$19/20 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1/20 & 0 \\ 0 & 19/20 \end{pmatrix}$	0.2161	0.0703	0.2864
3	10	90	$1/10 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$9/10 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 10 & 0 \\ 0 & 9/10 \end{pmatrix}$	0.3322	0.1368	0.4690
4	15	85	$3/20 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$17/20 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 3/20 & 0 \\ 0 & 17/20 \end{pmatrix}$	0.4105	0.1993	0.6098
5	20	80	$1/5 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$4/5\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1/5 & 0 \\ 0 & 4/5 \end{pmatrix}$	0.4644	0.2575	0.7219
6	25	75	$1/4 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$3/4\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1/4 & 0 \\ 0 & 3/4 \end{pmatrix}$	0.5	0.3113	0.8113
7	30	70	$3/10 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$7/10 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 3/10 & 0 \\ 0 & 7/10 \end{pmatrix}$	0.5211	0.3602	0.8813

8	35	65	7/10 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	13/10 $\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$	$\begin{pmatrix}7/10 & 0\\0 & 13/10\end{pmatrix}$	0.5301	0.4040	0.9341
9	40	60	2/5 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	3/5$\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$	$\begin{pmatrix}2/5 & 0\\0 & 3/5\end{pmatrix}$	0.5288	0.4422	0.9710
10	45	55	9/20 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	11/20 $\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$	$\begin{pmatrix}9/20 & 0\\0 & 11/20\end{pmatrix}$	0.5184	0.4744	0.9928
11	50	50	1/2 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	1/2$\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$	$\begin{pmatrix}1/2 & 0\\0 & 1/2\end{pmatrix}$	0.5	0.5	1
12	55	45	11/20 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	9/20 $\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$	$\begin{pmatrix}11/20 & 0\\0 & 9/20\end{pmatrix}$	0.4744	0.5184	0.9928
13	60	40	3/5 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	2/5$\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$	$\begin{pmatrix}3/5 & 0\\0 & 2/5\end{pmatrix}$	0.4422	0.5288	0.9710
14	65	35	13/20 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	7/20 $\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$	$\begin{pmatrix}13/20 & 0\\0 & 7/20\end{pmatrix}$	0.4040	0.5301	0.9341
15	70	30	7/10 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	3/10 $\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$	$\begin{pmatrix}7/10 & 0\\0 & 3/10\end{pmatrix}$	0.3602	0.5211	0.8813
16	75	25	3/4 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	1/4$\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$	$\begin{pmatrix}3/4 & 0\\0 & 1/4\end{pmatrix}$	0.3113	0.5	0.8113
17	80	20	4/5 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	1/5$\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$	$\begin{pmatrix}4/5 & 0\\0 & 1/5\end{pmatrix}$	0.2575	0.4644	0.7219
18	85	15	17/20 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	3/20 $\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$	$\begin{pmatrix}17/20 & 0\\0 & 3/20\end{pmatrix}$	0.1993	0.4105	0.6098
19	90	10	9/10 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	1/10 $\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$	$\begin{pmatrix}9/10 & 0\\0 & 1/10\end{pmatrix}$	0.1368	0.3322	0.4690
20	95	5	19/20 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	1/20 $\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$	$\begin{pmatrix}19/20 & 0\\0 & 1/20\end{pmatrix}$	0.0703	0.2161	0.2864
21	100	0	$\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	0$\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$	$\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	0	0	0

Table 2: Check for Validity of Sub-Additivity Theorem

Sr. No	$S(Q_{AB})$	$S(Q_A)+S(Q_B)$	$S(Q_{AB}) \leq S(Q_A)+S(Q_B)$	Difference Value
1	0	0	Satisfied	0
2	0.2864	0.2864	Satisfied	0
3	0.4690	0.469	Satisfied	0
4	0.6098	0.6098	Satisfied	0
5	0.7219	0.7219	Satisfied	0
6	0.8113	0.8113	Satisfied	0
7	0.8813	0.8813	Satisfied	0
8	0.9341	0.9341	Satisfied	0
9	0.9710	0.971	Satisfied	0
10	0.9928	0.9928	Satisfied	0
11	1	1	Satisfied	0
12	0.9928	0.9928	Satisfied	0
13	0.9710	0.971	Satisfied	0
14	0.9341	0.9341	Satisfied	0
15	0.8813	0.8813	Satisfied	0
16	0.8113	0.8113	Satisfied	0
17	0.7219	0.7219	Satisfied	0
18	0.6098	0.6098	Satisfied	0

19	0.4690	0.469	Satisfied	0
20	0.2864	0.2864	Satisfied	0
21	0	0	Satisfied	0

Table3: Calculation of Entropies for Bipartite System composed of |0> and |+> states

Sr. No	% of State 'A' or '|0>'	% of State 'B' or '|+>'	Q_A	Q_B	Q_{AB}	$S(Q_A)$	$S(Q_B)$	$S(Q_{AB})$
1	0	100	0 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	1 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	0	0	0
2	5	95	1/20 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	19/20 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}21/40 & 19/40\\19/40 & 19/40\end{pmatrix}$	0.2161	0.0703	0.1652
3	10	90	1/10 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	9/10 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}11/20 & 9/20\\9/20 & 9/20\end{pmatrix}$	0.3322	0.1368	0.2745
4	15	85	3/20 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	17/20 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}23/40 & 17/40\\17/40 & 17/40\end{pmatrix}$	0.4105	0.1993	0.3600
5	20	80	1/5 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	4/5 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}6/10 & 4/10\\4/10 & 4/10\end{pmatrix}$	0.4644	0.2575	0.4287
6	25	75	1/4 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	3/4 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}5/8 & 3/8\\3/8 & 3/8\end{pmatrix}$	0.5	0.3113	0.4838

7	30	70	3/10 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	7/10 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}13/20 & 7/20\\7/20 & 7/20\end{pmatrix}$	0.5211	0.3602	0.5271
8	35	65	7/20 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	13/20 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}27/40 & 13/40\\13/40 & 13/40\end{pmatrix}$	0.5301	0.4040	0.5598
9	40	60	2/5 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	3/5 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}7/10 & 3/10\\3/10 & 3/10\end{pmatrix}$	0.5288	0.4422	0.5828
10	45	55	9/20 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	11/20 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}29/40 & 11/40\\11/40 & 11/40\end{pmatrix}$	0.5184	0.4744	0.5964
11	50	50	1/2 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	1/2 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}3/4 & 1/4\\1/4 & 1/4\end{pmatrix}$	0.5	0.5	0.6009
12	55	45	11/20 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	9/20 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}31/40 & 9/40\\9/40 & 9/40\end{pmatrix}$	0.4744	0.5184	0.5964
13	60	40	3/5 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	2/5 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}8/10 & 2/10\\2/10 & 2/10\end{pmatrix}$	0.4422	0.5288	0.5828
14	65	35	13/20 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	7/20 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}33/40 & 7/40\\7/40 & 7/40\end{pmatrix}$	0.4040	0.5301	0.5598
15	70	30	7/10 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	3/10 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}17/20 & 3/20\\3/20 & 3/20\end{pmatrix}$	0.3602	0.5211	0.5271
16	75	25	3/4 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	1/4 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}7/8 & 1/8\\1/8 & 1/8\end{pmatrix}$	0.3113	0.5	0.4838

17	80	20	4/5 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	1/5 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}9/10 & 1/10\\1/10 & 1/10\end{pmatrix}$	0.2575	0.4644	0.4287
18	85	15	17/20 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	3/20 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}37/40 & 3/40\\3/40 & 3/40\end{pmatrix}$	0.1993	0.4105	0.3600
19	90	10	9/10 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	1/10 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}19/20 & 1/20\\1/20 & 1/20\end{pmatrix}$	0.1368	0.3322	0.2745
20	95	5	19/20 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	1/20 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}39/40 & 1/40\\1/40 & 1/40\end{pmatrix}$	0.0703	0.2161	0.1652
21	100	0	1 $\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	0 $\begin{pmatrix}1/2 & 1/2\\1/2 & 1/2\end{pmatrix}$	$\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$	0	0	0

Table 4: Validity Check of Sub-Additivity Theorem Inequality

Sr. No	Obtained $S(Q_{AB})$	$S(Q_A)+S(Q_B)$	$S(Q_{AB}) \leq S(Q_A)+S(Q_B)$	Difference Value
1	0	0	Satisfied	0
2	0.1652	0.2864	Satisfied	0.1212
3	0.2745	0.469	Satisfied	0.1945
4	0.3600	0.6098	Satisfied	0.2498
5	0.4287	0.7219	Satisfied	0.2932
6	0.4838	0.8113	Satisfied	0.3275
7	0.5271	0.8813	Satisfied	0.3542
8	0.5598	0.9341	Satisfied	0.3743

9	0.5828	0.971	Satisfied	0.3882
10	0.5964	0.9928	Satisfied	0.3964
11	0.6009	1	Satisfied	0.3991
12	0.5964	0.9928	Satisfied	0.3964
13	0.5828	0.971	Satisfied	0.3882
14	0.5598	0.9341	Satisfied	0.3743
15	0.5271	0.8813	Satisfied	0.3542
16	0.4838	0.8113	Satisfied	0.3275
17	0.4287	0.7219	Satisfied	0.2932
18	0.3600	0.6098	Satisfied	0.2498
19	0.2745	0.469	Satisfied	0.1945
20	0.1652	0.2864	Satisfied	0.1212
21	0	0	Satisfied	0

Table 5: Calculation of Entropies for Tripartite Quantum System

Sr. No	% of 'A'	% of 'B'	% of 'C'	Q_A	Q_B	Q_C
1	5	5	90	$\begin{pmatrix} 3/80 & 0 \\ 0 & 1/80 \end{pmatrix}$	$\begin{pmatrix} 9/200 & 0 \\ 0 & 1/200 \end{pmatrix}$	$\begin{pmatrix} 9/20 & 9/40 \\ 9/40 & 9/20 \end{pmatrix}$
2	5	45	50	$\begin{pmatrix} 3/80 & 0 \\ 0 & 1/80 \end{pmatrix}$	$\begin{pmatrix} 81/200 & 0 \\ 0 & 9/200 \end{pmatrix}$	$\begin{pmatrix} 1/4 & 1/8 \\ 1/8 & 1/4 \end{pmatrix}$
3	5	90	5	$\begin{pmatrix} 3/80 & 0 \\ 0 & 1/80 \end{pmatrix}$	$\begin{pmatrix} 81/100 & 0 \\ 0 & 9/100 \end{pmatrix}$	$\begin{pmatrix} 1/40 & 1/80 \\ 1/80 & 1/40 \end{pmatrix}$

4	10	30	60	$\begin{pmatrix}3/40 & 0\\0 & 1/40\end{pmatrix}$	$\begin{pmatrix}27/100 & 0\\0 & 3/100\end{pmatrix}$	$\begin{pmatrix}3/10 & 3/20\\3/20 & 3/10\end{pmatrix}$
5	10	10	80	$\begin{pmatrix}3/40 & 0\\0 & 1/40\end{pmatrix}$	$\begin{pmatrix}9/100 & 0\\0 & 1/100\end{pmatrix}$	$\begin{pmatrix}2/5 & 1/5\\1/5 & 2/5\end{pmatrix}$
6	10	80	10	$\begin{pmatrix}3/40 & 0\\0 & 1/40\end{pmatrix}$	$\begin{pmatrix}18/25 & 0\\0 & 2/25\end{pmatrix}$	$\begin{pmatrix}1/20 & 1/40\\1/40 & 1/20\end{pmatrix}$
7	25	35	40	$\begin{pmatrix}3/16 & 0\\0 & 1/16\end{pmatrix}$	$\begin{pmatrix}63/200 & 0\\0 & 7/200\end{pmatrix}$	$\begin{pmatrix}1/5 & 1/10\\1/10 & 1/5\end{pmatrix}$
8	30	60	10	$\begin{pmatrix}9/40 & 0\\0 & 3/40\end{pmatrix}$	$\begin{pmatrix}27/50 & 0\\0 & 3/50\end{pmatrix}$	$\begin{pmatrix}1/20 & 1/40\\1/40 & 1/20\end{pmatrix}$
9	35	40	25	$\begin{pmatrix}21/80 & 0\\0 & 7/80\end{pmatrix}$	$\begin{pmatrix}9/25 & 0\\0 & 1/25\end{pmatrix}$	$\begin{pmatrix}1/8 & 1/16\\1/16 & 1/8\end{pmatrix}$
10	40	25	35	$\begin{pmatrix}3/10 & 0\\0 & 1/10\end{pmatrix}$	$\begin{pmatrix}9/40 & 0\\0 & 1/40\end{pmatrix}$	$\begin{pmatrix}7/40 & 7/80\\7/80 & 7/40\end{pmatrix}$
11	45	50	5	$\begin{pmatrix}27/80 & 0\\0 & 9/80\end{pmatrix}$	$\begin{pmatrix}9/20 & 0\\0 & 1/20\end{pmatrix}$	$\begin{pmatrix}1/40 & 1/80\\1/80 & 1/40\end{pmatrix}$
12	50	5	45	$\begin{pmatrix}3/8 & 0\\0 & 1/8\end{pmatrix}$	$\begin{pmatrix}9/200 & 0\\0 & 1/200\end{pmatrix}$	$\begin{pmatrix}9/40 & 9/80\\9/80 & 9/40\end{pmatrix}$
13	60	10	30	$\begin{pmatrix}9/20 & 0\\0 & 3/20\end{pmatrix}$	$\begin{pmatrix}9/100 & 0\\0 & 1/100\end{pmatrix}$	$\begin{pmatrix}3/20 & 3/40\\3/40 & 3/20\end{pmatrix}$
14	80	10	10	$\begin{pmatrix}3/5 & 0\\0 & 1/5\end{pmatrix}$	$\begin{pmatrix}9/100 & 0\\0 & 1/100\end{pmatrix}$	$\begin{pmatrix}1/20 & 1/40\\1/40 & 1/20\end{pmatrix}$
15	90	5	5	$\begin{pmatrix}27/40 & 0\\0 & 9/40\end{pmatrix}$	$\begin{pmatrix}9/200 & 0\\0 & 1/200\end{pmatrix}$	$\begin{pmatrix}1/40 & 1/80\\1/80 & 1/40\end{pmatrix}$

Table 5 continued…

Sr. No	Q_{AB}	Q_{BC}	Q_{ABC}	S (Q_B)	S (Q_{AB})	S (Q_{BC})	S(Q_{ABC})
1	$\begin{pmatrix}0.0825 & 0\\0 & 0.0175\end{pmatrix}$	$\begin{pmatrix}0.4950 & 0.2250\\0.2250 & 0.4550\end{pmatrix}$	$\begin{pmatrix}0.5325 & 0.2250\\0.2250 & 0.4675\end{pmatrix}$	0.2678	0.3991	0.8589	0.8453
2	$\begin{pmatrix}0.4425 & 0\\0 & 0.5750\end{pmatrix}$	$\begin{pmatrix}0.6550 & 0.1250\\0.1250 & 0.2950\end{pmatrix}$	$\begin{pmatrix}0.6925 & 0.1250\\0.1250 & 0.3075\end{pmatrix}$	0.7294	0.9796	0.8588	0.8421

3	$\begin{pmatrix} 0.8475 & 0 \\ 0 & 0.1025 \end{pmatrix}$	$\begin{pmatrix} 0.8350 & 0.0125 \\ 0.0125 & 0.115 \end{pmatrix}$	$\begin{pmatrix} 0.8725 & 0.0125 \\ 0.0125 & 0.1275 \end{pmatrix}$	0.5589	0.5392	0.5754	0.5500
4	$\begin{pmatrix} 0.3450 & 0 \\ 0 & 0.0550 \end{pmatrix}$	$\begin{pmatrix} 0.5700 & 0.1500 \\ 0.1500 & 0.3300 \end{pmatrix}$	$\begin{pmatrix} 0.645 & 0.1500 \\ 0.1500 & 0.3550 \end{pmatrix}$	0.6618	0.7598	0.9146	0.8705
5	$\begin{pmatrix} 0.1650 & 0 \\ 0 & 0.0350 \end{pmatrix}$	$\begin{pmatrix} 0.4900 & 0.2000 \\ 0.2000 & 0.4100 \end{pmatrix}$	$\begin{pmatrix} 0.5650 & 0.2000 \\ 0.2000 & 0.435 \end{pmatrix}$	0.3791	0.5982	0.8984	0.8683
6	$\begin{pmatrix} 0.7950 & 0 \\ 0 & 0.1050 \end{pmatrix}$	$\begin{pmatrix} 0.7700 & 0.0250 \\ 0.0250 & 0.1300 \end{pmatrix}$	$\begin{pmatrix} 0.8450 & 0.0375 \\ 0.0375 & 0.1550 \end{pmatrix}$	0.6942	0.8263	0.9430	0.8474
7	$\begin{pmatrix} 0.5025 & 0 \\ 0 & 0.0975 \end{pmatrix}$	$\begin{pmatrix} 0.5150 & 0.1000 \\ 0.1000 & 0.2350 \end{pmatrix}$	$\begin{pmatrix} 0.7027 & 0.1000 \\ 0.1000 & 0.2975 \end{pmatrix}$	0.6327	0.6045	0.6705	0.6172
8	$\begin{pmatrix} 0.7650 & 0 \\ 0 & 0.3150 \end{pmatrix}$	$\begin{pmatrix} 0.5900 & 0.0250 \\ 0.0250 & 0.1100 \end{pmatrix}$	$\begin{pmatrix} 0.8150 & 0.0250 \\ 0.0250 & 0.8150 \end{pmatrix}$	0.7236	0.6857	0.7962	0.6888
9	$\begin{pmatrix} 0.6225 & 0 \\ 0 & 0.1275 \end{pmatrix}$	$\begin{pmatrix} 0.4850 & 0.0625 \\ 0.0625 & 0.1650 \end{pmatrix}$	$\begin{pmatrix} 0.7475 & 0.0625 \\ 0.0625 & 0.2525 \end{pmatrix}$	0.7164	0.8046	0.9161	0.8028
10	$\begin{pmatrix} 0.5250 & 0 \\ 0 & 0.1250 \end{pmatrix}$	$\begin{pmatrix} 0.4000 & 0.0875 \\ 0.0875 & 0.2000 \end{pmatrix}$	$\begin{pmatrix} 0.7000 & 0.0875 \\ 0.0875 & 0.3000 \end{pmatrix}$	0.6172	0.8630	0.9543	0.8578
11	$\begin{pmatrix} 0.7875 & 0 \\ 0 & 0.1625 \end{pmatrix}$	$\begin{pmatrix} 0.4750 & 0.0125 \\ 0.0125 & 0.0750 \end{pmatrix}$	$\begin{pmatrix} 0.8125 & 0.0125 \\ 0.0125 & 0.1875 \end{pmatrix}$	0.7345	0.6974	0.7894	0.6957
12	$\begin{pmatrix} 0.4200 & 0 \\ 0 & 0.1300 \end{pmatrix}$	$\begin{pmatrix} 0.2700 & 0.1125 \\ 0.1125 & 0.2300 \end{pmatrix}$	$\begin{pmatrix} 0.6450 & 0.1125 \\ 0.1125 & 0.3550 \end{pmatrix}$	0.2395	0.9083	0.9218	0.9005
13	$\begin{pmatrix} 0.5400 & 0 \\ 0 & 0.1600 \end{pmatrix}$	$\begin{pmatrix} 0.2400 & 0.0750 \\ 0.0750 & 0.1600 \end{pmatrix}$	$\begin{pmatrix} 0.6900 & 0.0750 \\ 0.0750 & 0.3100 \end{pmatrix}$	0.3791	0.9031	0.8750	0.8760
14	$\begin{pmatrix} 0.7350 & 0 \\ 0 & 0.2150 \end{pmatrix}$	$\begin{pmatrix} 0.1400 & 0.0250 \\ 0.0250 & 0.0600 \end{pmatrix}$	$\begin{pmatrix} 0.7400 & 0.0250 \\ 0.0250 & 0.2600 \end{pmatrix}$	0.3791	0.8033	0.6310	0.8248
15	$\begin{pmatrix} 0.7200 & 0 \\ 0 & 0.2300 \end{pmatrix}$	$\begin{pmatrix} 0.0700 & 0.0125 \\ 0.0125 & 0.0300 \end{pmatrix}$	$\begin{pmatrix} 0.7450 & 0.0125 \\ 0.0125 & 0.2550 \end{pmatrix}$	0.2395	0.8289	0.4155	0.8186

Table 6: Strong Sub-Additivity Inequality Theorem Check

Sr. No	$S(Q_{ABC})+S(Q_B)$	$S(Q_{AB})+S(Q_{BC})$	$S(Q_{ABC})+S(Q_B) \leq S(Q_{AB})+S(Q_{BC})$	Difference Value
1	1.1131	1.2580	Satisfied	0.1449
2	1.5715	1.8384	Satisfied	0.2669

3	1.1089	1.1146	Satisfied	0.0057
4	1.5323	1.6744	Satisfied	0.1421
5	1.2474	1.4966	Satisfied	0.2492
6	1.5416	1.7693	Satisfied	0.2277
7	1.2499	1.275	Satisfied	0.0251
8	1.4124	1.4819	Satisfied	0.0695
9	1.5192	1.7207	Satisfied	0.2015
10	1.4750	1.8173	Satisfied	0.3423
11	1.4302	1.4868	Satisfied	0.0566
12	1.1400	1.8301	Satisfied	0.6901
13	1.2551	1.7781	Satisfied	0.5230
14	1.2039	1.4343	Satisfied	0.2304
15	1.0581	1.2444	Satisfied	0.1863

The following are the plots that were obtained for bipartite systems

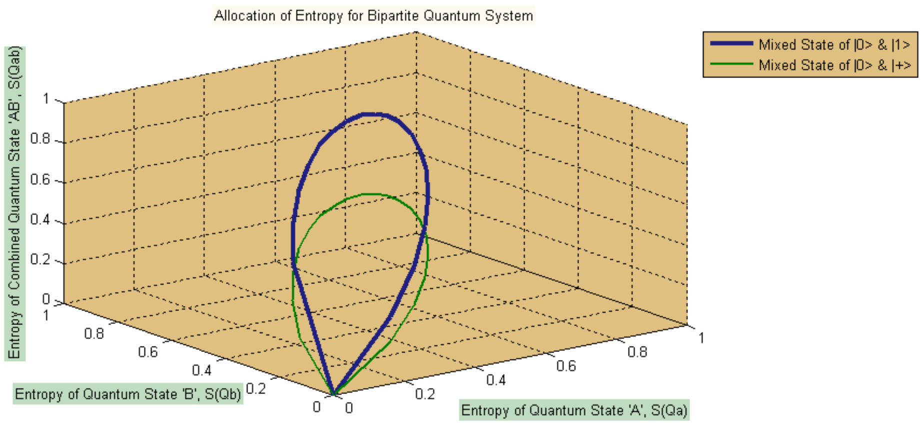

Figure 1: Allocation of Entropy for bipartite system of |0> and |1>

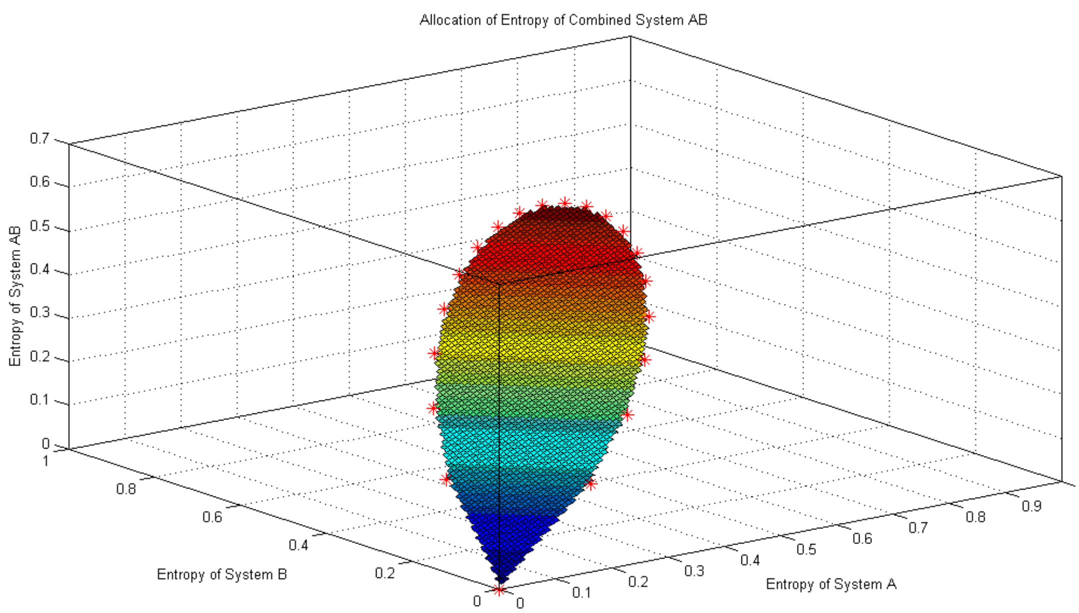

Figure 2: Allocation of Entropy for Mixed state of (i)|0> and |1> (shown in blue color) and (ii) |0> and |+> (shown in green color)

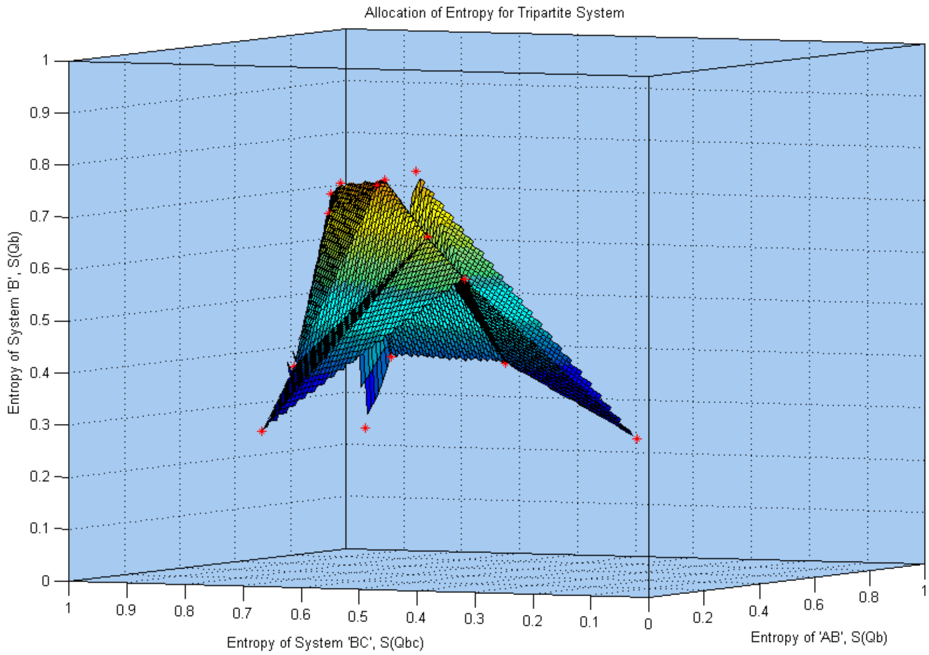

Figure 3: Allocation of Entropy for Tri-Partite System

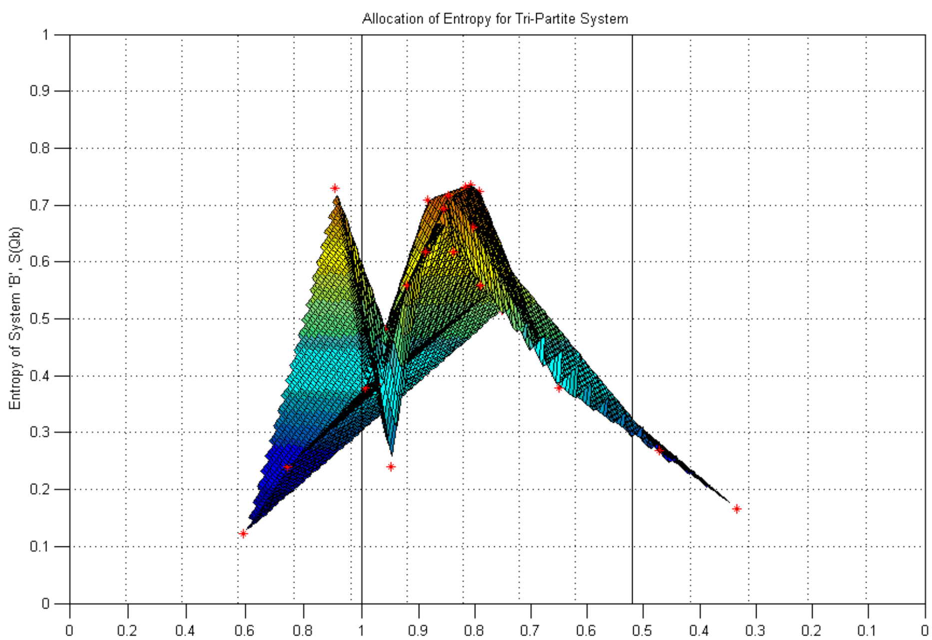

Figure 4: Allocation of Entropies for Tri-Partite System

6. Conclusion

Thus as per the objective of this project, the allocation of entropies for bipartite and tripartite quantum system were plotted and pictorial view obtained using Matlab. The geometrical gave lot of information about the quantum system through entropy. For example where and at what values entropy is maximized by simultaneously considering all separate system and the combined system. The validity of the Inequalities of Quantum Information Theory has been checked and verified for the two types of combined system and thus the realization of the Inequalities of QIT has been successfully achieved in Matlab

7. References

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